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ABSTRACT

This resource book for algebra contains games, tricks, and quick activities for the classroom. Categories of activities include puzzlers, patterns, manipulatives, measurement, graphing, and a section that contains reproducible statement and value cards. Twenty one puzzle problems, four pattern activities, and 11 quick activities that engage students in graphing are provided. Nine manipulative activities that use simple supplies and four measurement problems are also included. Solutions to all the activities are provided at the end of the book. (DDR)

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SCRATCH YOUR BRAIN WHERE IT ITCHES

BOOK D-1 ALGEBRA

MATH GAMES, TRICKS & QUICK ACTIVITIES

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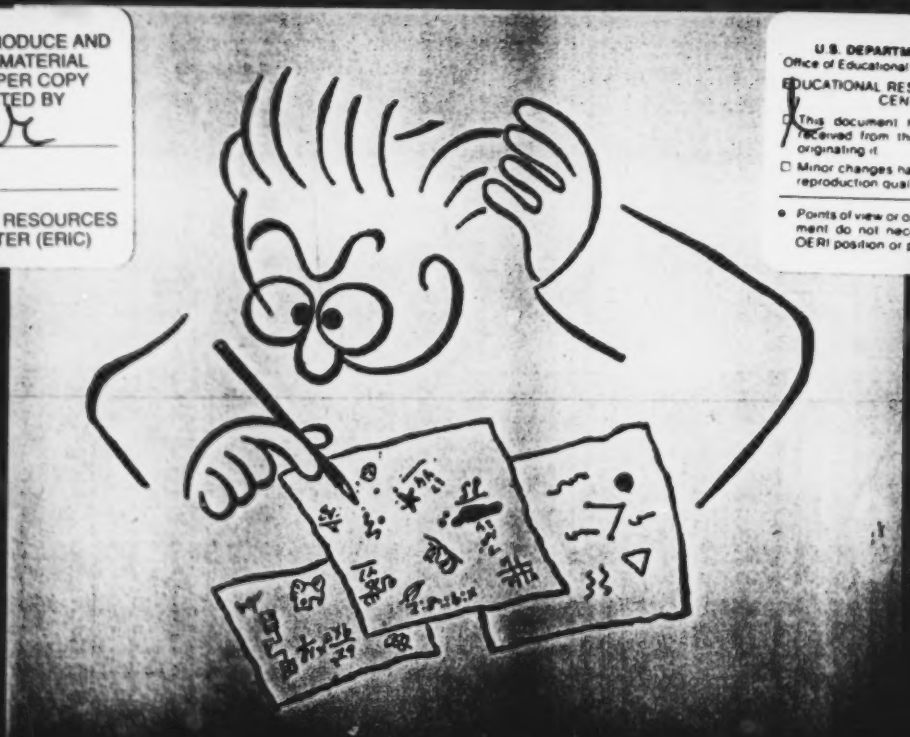

DOUG BRUMBAUGH

CRITICAL THINKING PRESS & SOFTWARE

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Scratch Your Brain Where It Itches

D-1

Algebra Games, Tricks, and Quick Activities

DOUG BRUMBAUGH

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PUZZLERS

1. Roll three die. Select the value from one of the three. Multiply that value by 2. Add 5 to the product. Multiply the sum by 5. Add the value of one of the two remaining die. Multiply this new sum by 10. Add the value from the remaining die. Subtract 250 from this new sum. The result will be the three original die values. Use variables to show how this problem works and write a paragraph explaining your solution.

2. Write your phone number. Multiply that value by 2. Add 5. Multiply the sum by 50. Add your age to the product. Add 365 to the new sum. Subtract 615 from this sum. The result will be your phone number and age, with your age being in the ones and tens place of the result. Use variables to show how this problem works, and write a paragraph explaining your solution.

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3. $(x - a)\ (x - b)\ (x - c)\ \ldots\ (x - y)\ (x - z) = ????$

4. Pick a number. Double it. Add 4 to the result. Divide the sum by 2. Subtract the original number. What do you get? Use variables to show how this problem works, and write a paragraph explaining your solution.

EXTENSION: How could the instructions be altered to get a different answer?

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5. Add WHICH to WHAT and divide by 4;
WHICH squared is now in store.
WHAT to WHICH is 35 to 1, you see;
The values of each, please tell me.

EXTENSION: Create a similar problem statement and solution.

6. $1.65 is made up of pennies, nickels and dimes. Half of the coins are nickels. List a set of coins that will solve this problem.

EXTENSION: Give a formula for the solution of this problem. List all possible solutions.

EXTENSION: Make up a problem like this that cannot be solved.

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7. Pick any prime number greater than 3. Square it. Add 15. Divide by 12. What is the remainder?

EXTENSION: Prove why this is so.

8. If $\triangle x = x + 3$ and $\boxed{x} = x^2 - 4$, find x such that

$x^2 - \boxed{\triangle x} = 5$.

EXTENSION: Make up a similar problem and give it to a classmate to solve. Write a description of how the problem is solved.

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9. Take any 3-digit number. Repeat the selected number, writing the second beside the first, making a 6-digit number. Divide that 6-digit number by 7, 11, and 13. The final missing factor will be the original 3-digit number. Write an explanation of how this works.

10. Pick any counting number. Square it. Square the counting number that is one larger than your initial selection. Find the difference between the two squares. Subtract one from the difference. Divide the difference by two. What do you get? Will this work every time? Why or why not?

EXTENSION: Does this work with integers? rational numbers?

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11. Pick a counting number. Square it. Take the next largest counting number. Square it. Find a fast way to find the second square.

EXTENSION: Will this work for integers? fractions? decimals?

12. Two objects have a combined price of $6.80. One of the objects sells for $6.00 more than the other. What is the cost of the less expensive object (it is not $0.80)?

EXTENSION: Create a similar problem. Explain in writing the similarities and differences between your problem and this one and how they both work.

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13. Solve the following and time yourself.

$$\frac{1234567890}{(1234567891)^2 - (1234567890)\,(1234567892)}$$

EXTENSION: Can you find a faster way to do this?

14. Select 3 digits; do not repeat the digits. Form all possible 2-digit numbers using these 3 digits. Find the sum of all six 2-digit numbers. To find this total quickly, sum the original 3 digits, double this sum, and multiply the result by 11. This product is the sum of all six 2-digit numbers. How does this technique work?

EXTENSION: Do the same thing, forming all possible 3-digit combinations of the initial 3 digits.

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15. Given a quadratic expression $ax^2 + bx + c$, the variables a, b, and c can always be determined by the following process:
Substitute 0 into the expression.
Record the result (call it r).
Substitute 1 into the expression.
Record the result (call it s).
Substitute 2 into the expression.
Record the result (call it t).
Record the result of $s - r$ (call it u).
Record the result of $t - s$ (call it v).
Record the result of $v - u$ (call it w).
In the quadratic equation, $a = 0.5w$, $b = u - 0.5w$, $c = r$.
Ask someone to use a quadratic expression and give you r, s, and t. You can then identify the original quadratic expression used.

EXTENSION: How does this work?

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16. Let $a = b$.
Multiply both sides of the equation by b: $ab = b^2$.
Subtract a^2 from both sides: $ab - a^2 = b^2 - a^2$.
Factor both sides: $a(b - a) = (b + a)(b - a)$.
Divide by common factor: $a = a + b$.
Substitute a for b: $a = a + a$.
Simplify: $a = 2a$.
Divide by the common factor: $1 = 2$.
Find the error that led to this wrong conclusion.

EXTENSION: Find other examples like this one.

17. How can you tell quickly if a number is divisible by 2? Show why this rule works.

EXTENSION: Rules for divisibility by whole-number powers of 2 (4, 8, 16, ...) are similar (but after 8, there is a question of practicality). Show how these rules work. The rule for 5 is similar. Write a description of some common characteristics of these rules.

18. How can you tell quickly if a number is divisible by 3? Show how this rule works.

EXTENSION: Describe a similar situation for divisibility by 9.

19. Would you chose to make $10,000 a day every day for a month or a penny the first day, $0.02 the second day, $0.04 the third day, and so on, doubling the amount from the previous day for a month?

EXTENSION: Using both pay methods, how much more would you make in January than in February (don't forget leap year)?

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20. What is the largest number that can be written using 3 digits?

21. What is the answer to 2^{3^4}? Is this computed from the top down or the bottom up? That is, is it 2^{81} or is it 8^4?

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22. Fibonacci's sequence 1, 1, 2, 3, 5, 8, 13, ... is generated by adding the two preceding terms to get any given term. Any two starting values can be selected to generate a Fibonacci sequence. Determine how the sum of the first ten terms in a Fibonacci sequence can quickly be found. Use variables to show how this problem works and write a paragraph explaining your solution.

NOTE: Any values may be used to start with, but small counting numbers, particularly at first, should be selected to simplify the arithmetic.

EXTENSION: Would 20 terms have the 14th term multiplied by 11 or the 7th term multiplied by 22?

EXTENSION: Find some applications of Fibonacci's sequence in nature. Find other patterns similar to the solution of this question.

23. (2) (2) = 4 = 2 + 2. This is not the only example of this relationship. There are an infinite number of such pairs. Find at least two more. One of the examples could be pretty basic.

EXTENSION: Show how an infinite number of such pairs could be determined. What is the largest set of numbers the generalization works with (counting numbers, whole numbers, integers, rational numbers, real numbers, complex numbers, quaternions)?

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24. This situation involves squaring numbers that end in 5. You are given the answers for some examples below. Most of the examples are for two-digit numbers, but you see it works for one- and three-digit numbers, too. Your task is to explain how the pattern works. There is more than one way to do it.

$5^2 = 25$
$15^2 = 225$
$25^2 = 625$
$35^2 = 1225$
$45^2 = 2025$
$55^2 = 3025$
$65^2 = 4225$
$75^2 = 5625$
$95^2 = 9025$
$125^2 = 15625$

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25. The products below can be done quickly in your head. How?

(36) (34) = 1224
(53) (57) = 3021
(62) (68) = 4216
(41) (49) = 2009

EXTENSION: Write a description of how the rule you devised works. Prove why your rule works (HINT—see the previous problem).

26. In the two examples below, reversing the order of the digits in both factors gives the same product. Find other examples where this is true.

(36) (42) = 1512 = (63) (24)
(26) (93) = 2418 = (62) (39)

EXTENSION: Find other examples. Show when this will be true.

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27. Pick any counting number.
Add the next highest counting number.
Add 9 to the sum.
Divide this new sum by 2.
Subtract 5.
What do you get? How does this work?

EXTENSION: Will this work for integers? rational numbers?

28. Pick any counting number.
Multiply it by 6.
Add 12 to that product.
Divide the new sum by 3.
Subtract 2 from the result (missing factor).
Divide this difference by 2.
Subtract the original number from this missing factor.
Add 9 to the difference.
What is the sum? Why will the sum always be the same?

EXTENSION: Does this work for integers? rational numbers?

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29. When Johann Friederich Carl Gauss was 10, he was asked by his teacher to find the sum of the "following sort: 81297 + 81495 + 81693 + ... + 100899, where the step from one number to the next is the same all along [here 198] and a given number of terms [here 100]."[1]

Gauss had the sum almost as quickly as the problem was finished being stated while the rest of the class took over an hour to do it. A similar problem like 1 + 2 + 3 + ... + 100 can be used to show the basics of what Gauss did to solve the problem. To get the sum, multiply (100)(101), or the number of terms (100) times the answer from adding the first and last term (100 + 1 = 101). However, this sum is twice what it should be to answer the problem because of using each addend twice. So the product 10,100 should be divided by two, giving 5050, the sum of the first 100 consecutive counting numbers. Do a similar problem where the constant difference between terms is not one, trying to determine a rule for the solution.

EXTENSION: Give a generalization showing how to do the sum of the first *n* consecutive counting numbers. Develop a generalization that would work for the type of problem Gauss did.

[1]Newman, James R. *The World of Mathematics*. New York: Simon and Schuster, 1956, page 298.

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30. A volunteer selects a square of any 9 dates from any month of a calendar. The smallest value in the square is announced. You can now tell the sum of all 9 dates in the square. How does this work?

EXTENSION: How would this work for a 16-date square? 25-date square? Describe a generalization.

31. To find out whether a number is divisible by 7, double the last digit in the original value. Delete the last digit in the original value. Subtract the double of the last digit from the new value created by deleting the last digit in the original value. If this new value is divisible by 7, the original value is also. If this new value is not divisible by 7, the original is not. If you do not know if the new value is divisible by 7, the process may be repeated as many times as needed. (In the example below, 28 is divisible by 7, so 39823 is divisible by 7.) What is the logic behind how this rule works? Is 2296 divisible by 7?

EXTENSION: Show why the rule works.

39823
- 6
3976
-12
384
10
28

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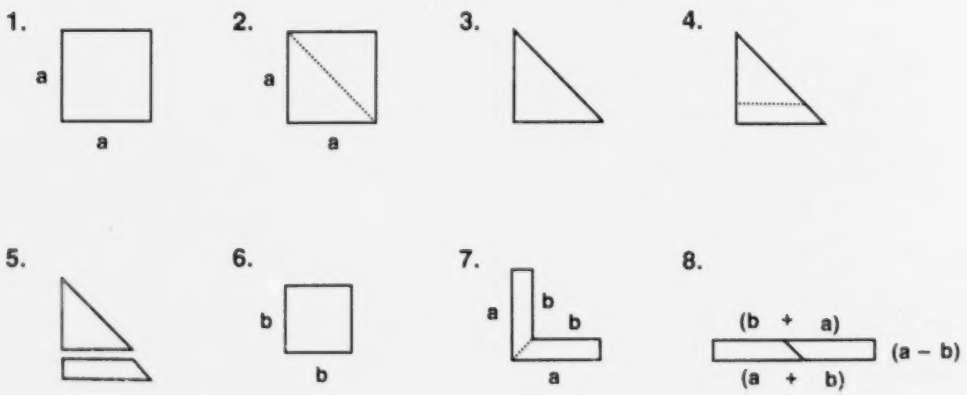
32. Make a square piece of paper (figure 1). Define the side length to be a. The area of the paper is a^2. Fold the square along a major diagonal (figure 2). Describe the resulting figure (figure 3). Make a fold so the new fold is parallel to one of the legs (figure 4). Cut along the new fold line. Consider the created triangle first, placing the trapezoid off to the side (figure 5). Opening the triangle yields another square (figure 6). Define the length of this side to be b, so the area of this new square is b^2. Open the trapezoid to reveal an L-shaped hexagon (figure 7). This represents the area $a^2 - b^2$. Label the long sides of the "L" as a and the sides parallel to them as b. What is the length of the sides at the top and end of the "L" which are perpendicular to both a and b on their respective sides? Cut the "L" along the fold line, then flip and rotate one of the pieces. Join the two pieces of the "L" back together along the diagonal cut, forming a rectangle (figure 8). Define the length and width of this new rectangle. Using the dimensions of the rectangle, express the area of that rectangle. This area is the same as the area of the "L" piece expressed earlier as $a^2 - b^2$. Describe the conclusions.

EXTENSION: Describe concrete approaches for finding $(a + b)^2$ and $(a - b)^2$.



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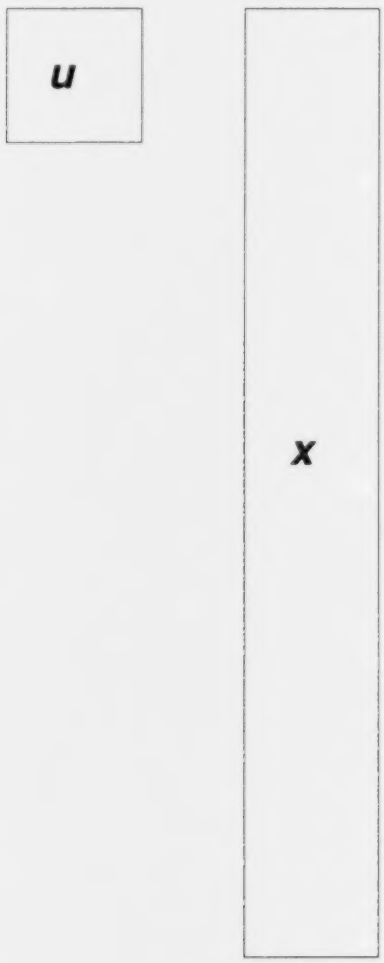
33. Make a unit square and a rectangle that is as wide as the square but much longer. Call the square u and the rectangle x. Use them to show multiplication. Suppose you have 4 sets, each of which has 2 x's and 3 u's. Collecting like terms, you get 8 x's and 12 u's, which is the same as $4(2x+3u)$. This can be shown with the pieces as a rectangle that is made up of four rows, each of which is $2x + 3u$ long. Notice the dimensions of the rectangle are the factors of the problem, and the area of the rectangle is the product of those factors.

EXTENSION: Make up several problems and build the rectangles that represent the respective products.



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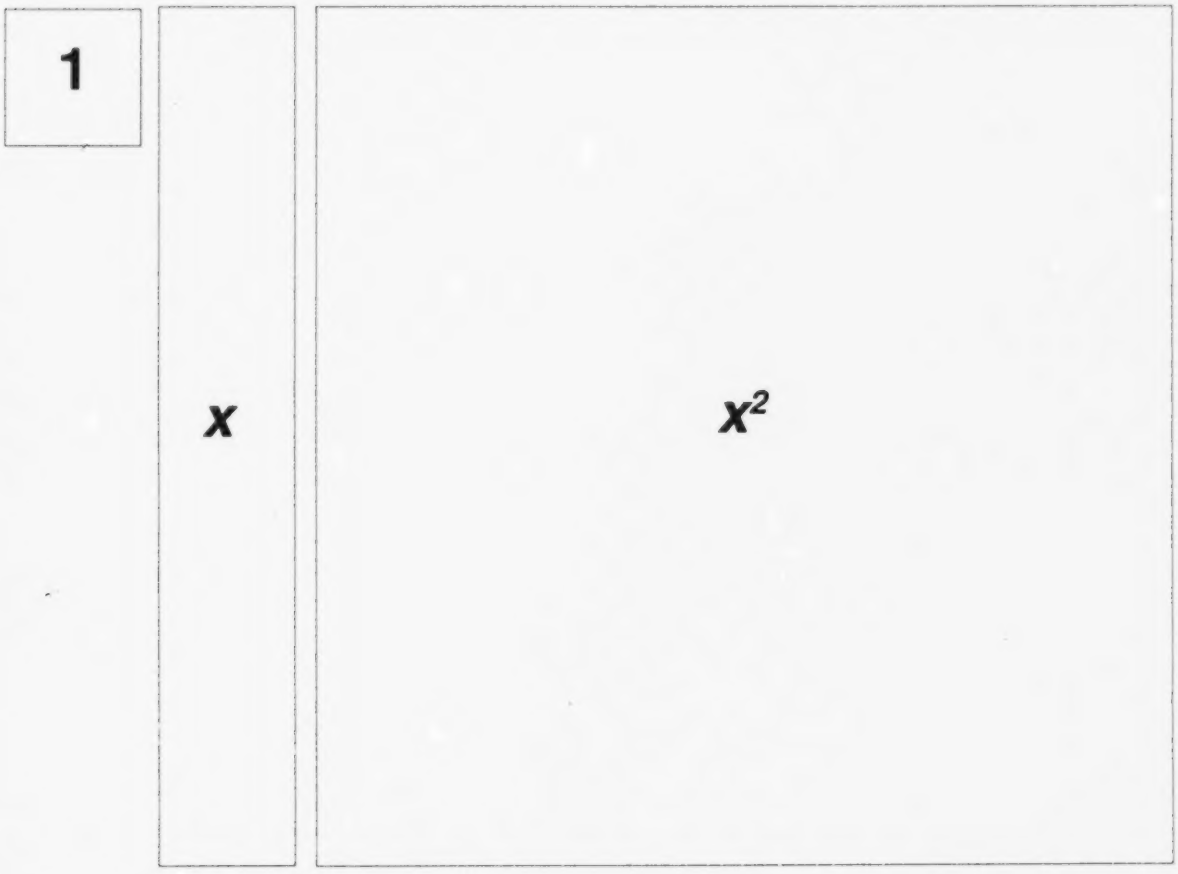
34. Define a unit square, a rectangle that is the unit wide and some multiple of the unit long, and a large square that has its side length equal to that of the length of the rectangle. Define these three pieces as 1, x, and x^2, respectively. Make several of each of these pieces, having more of the 1s than x's and more x's than x^2s. Use the following pieces to build a rectangle: one x^2, three x's, and two 1s—$(x^2 + 3x + 2)$. Describe the length and width of the rectangle.

NOTE: An x cannot be traded for the number of 1s equal to its length. A similar rule applies for x and x^2.

EXTENSION: Build other figures and determine the polynomials that the figures represent. Generalize about which polynomials can be constructed and which cannot.



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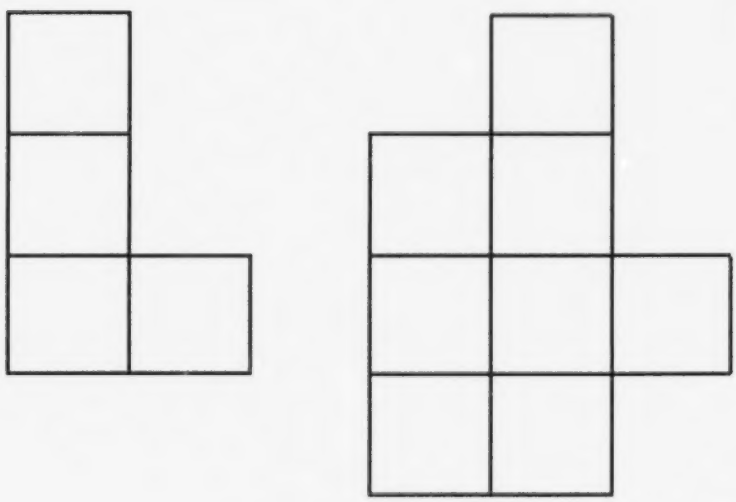
35. Make an "L" shaped figure from 4 congruent squares. The "L" should be 3 squares high and 2 squares wide, as shown. What is the perimeter of this figure? Place two "L's" together, inside each other, as shown. What is the perimeter of this figure? If 100 "L's" were placed together like this, what would the perimeter be?

EXTENSION: Design a different figure; place several of them together in some consistent manner and determine a formula for the perimeter as the number of pieces changes.



36. Cut a piece of paper into 4 pieces. Cut one of those 4 pieces into 4 more pieces. Repeat the process. After *n* repetitions, how many pieces of paper will you have?

EXTENSION: Use a value different from 4 and answer the questions. Repeat with a different number. Describe in writing any patterns observed.

37. Suppose a sheet of notebook paper is 0.003 inches thick. Suppose you could fold that piece of paper in half 50 times. How high would the stack be? If the paper were 0.004 inches thick, how high would the stack be?

EXTENSION: Determine the maximum number of times a piece of paper, no matter the size or type paper, can be physically folded in half.

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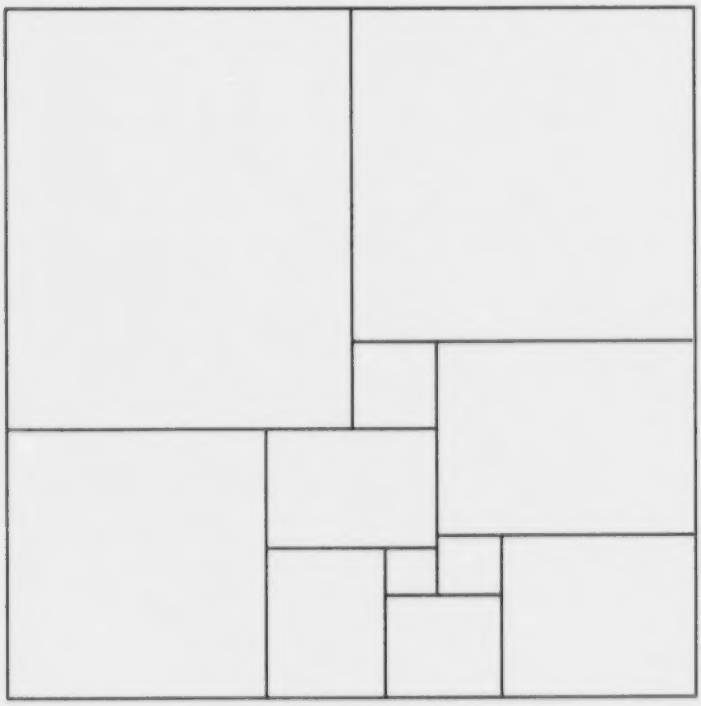
26. The rectangle below has been divided into eleven squares. No two squares have the same side length. The area of the smallest square is one. What is the area of the rectangle?



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39. Shuffle a standard rummy deck of cards. Create stacks of cards by turning one card over and, using that value to start, count one card at a time until you reach 13. Ignore the face value of the cards being turned over after the first one. For example, if you turn over a 7 for your first card, count the next 6 cards as "8," "9," "10," "11," "12," and "13" no matter what the cards are. Then you start a new stack. If you start with a face card, Jack = 11, Queen = 12, King = 13, and Ace = 1. Continue until you cannot make any more stacks. Hold any excess cards. Turn each stack face down, being careful that the first card for that stack is on top. Have someone remove all but 3 stacks, giving you the cards they eliminate. Reveal the top card on two of the stacks. By counting, you can tell the hidden card on top of the third stack. To do this, add the two card values showing to ten. Count through the cards in your hand until you get to that sum. Set those cards aside, then start counting from one again. The number of cards you have left in your hand will be the value of the hidden card. How does this work?

40. Using a standard rummy deck of cards, let Ace = 1, Jack = 11, Queen = 12, and King = 13. Also, Clubs = 4, Diamonds = 5, Hearts = 6, and Spades = 7. A volunteer should do each of the following: Select a card, show it to the class (but not to you), and return the card to the deck. Add the card value and the next highest counting number. Multiply the sum by 5. Add the value of the suit to this product. Tell the answer. At this point, you look through the deck and find the initial card. How does this work?

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41. A truck travels 25 mph for the first half of a trip. How fast must it travel in the second half of the trip to average 50 mph for the entire trip?

EXTENSION: Create similar problems.

42. The length of a log is 20-units plus half of its own length. How long is the log?

EXTENSION: Create a similar problem.

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43. Equations appear in this grid horizontally, vertically, and diagonally. Insert +, −, ×, ÷, and * (exponent) to form as many equations as you can.

8	2	64	1	1	3	7	1
6	4	3	1	27	1	4	0
5	2	32	1	27	1	4	0
9	1	2	8	25	27	8	31
4 * 3 = 64			6	2	3	36	6
7	16	4	8	2	1	49	16
1	18	36	7	2	2	3	27
64	9	4	8	32	1	25	1

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44. To establish a speed trap similar to those used by police, measure a distance (100 yards is convenient) along a straight section of road. Mark the beginning and end of the trap well. Place a person (with all due considerations for safety) at each end of the trap to indicate when a car enters and exits the trap (be sure both use the same end of the car — like the front bumper). Use a stopwatch and record the time required for each car to get through the trap to the nearest one hundredth of a second. Compute the average speed of each car, rounding to miles per hour. You know the distance and rate at the selected location; compute the legal time to the nearest hundredth of a second. Determine how much over or under the legal speed each timed car was traveling.

EXTENSION: Find another application of this procedure.

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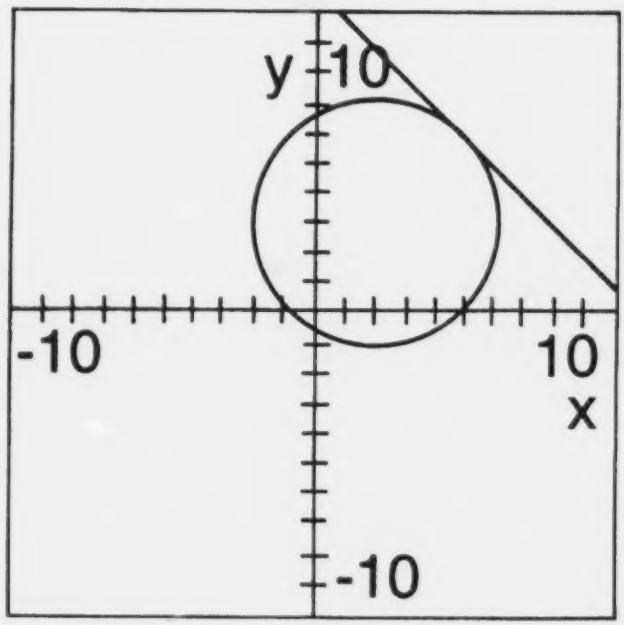
45. Use a graphing calculator or software to graph $(x-2)^2+(y-3)^2=16$ and $y=-x+10.8$. Although the graph of the line appears to be tangent to the circle, it is not. Use the zoom function to show this is, in fact, the case.

Substitute $-x+10.8$ for y in $(x-2)^2+(y-3)^2=16$ and solve for x. What do you get?

EXTENSION: Explain how the zoom mode of the graph is related to the answer derived when $-x+10.8$ is substituted for y in $(x-2)^2+(y-3)^2=16$.



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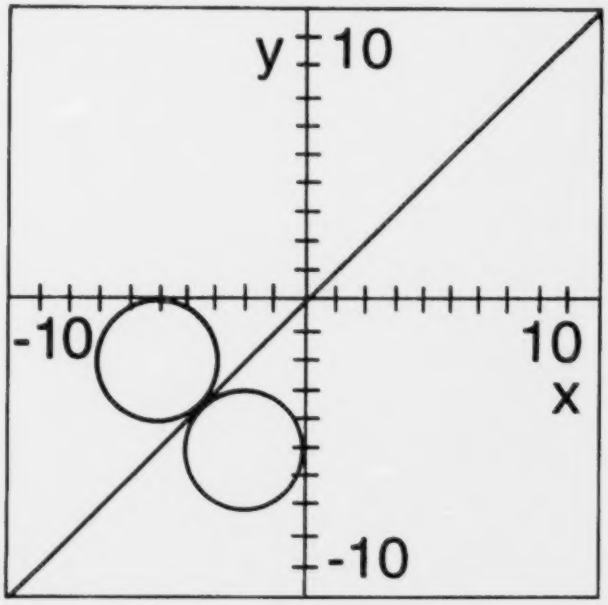
46. Use a graphing calculator or software to show, in more than one way, that the circles centered at (–2, –5) and (–5, –2), each with radius 2, are not tangent to the line $y = x$.



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47. Use a graphing calculator or software that will deal with algebraic expressions. Type in MISSISSIPPI (all capital letters) and simplify. Now type in mississippi (all lower case letters) and simplify. How are the two answers different? How are the two answers the same? Explain the differences.

EXTENSION: Create similar problems.

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48. Use a graphing calculator or software to graph the following; in each case, find the coordinates of the lowest point:

$y = x^2$
$y = x^2 + 1$
$y = x^2 + 2$
$y = x^2 + 3$
$y = x^2 + 4$

Predict what $y = x^2 + 6$ will look like. What will the coordinates of the lowest point be? Describe the graph and coordinates of the lowest point of any equation like $y = x^2 + a$ (where a is any counting number).

EXTENSION: Do the same activity where a is any integer. What happens if $y = -x^2 + a$ (where a is any integer)?

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49. Use a graphing calculator or software to graph the following; in each case, find the coordinates of the lowest point:

$y = x^2$
$y = (x + 1)^2$
$y = (x + 3)^2$
$y = (x + 5)^2$

Predict what $y = (x + 7)^2$ will look like. What are the coordinates of the lowest points on the graph? Describe the graph and coordinates of the lowest point of any equation like $y = (x + a)^2$ (where a is any counting number).

EXTENSION: Do the same activity where a is any integer. What happens if $y = -(x + a)^2$ (where a is any integer)?

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50. Use a graphing calculator or software to graph the following; in each case, find the coordinates of the lowest point:

$y = x^2$
$y = (x + 3)^2 + 4$
$y = (x + 5)^2 + 3$

Predict the graph of $y = (x + 7)^2 + 9$. What are the coordinates of the lowest point on the graph? Describe the graph and coordinates of the lowest point of any equation like $y = (x + a)^2 + b$ (where a and b are any counting numbers).

EXTENSION: Do the same activity where a and b are any integers. What happens if $y = -(x + a)^2 + b$ (where a and b are any integer)?

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51. Use a graphing calculator or software to show $f(g(4))$ does not equal $g(f(4))$ when $f(x) = 3x^2 - 5x - 7$ and $g(x) = x^2 + 3x - 4$.

EXTENSION: Do similar problems with different, more complex definitions of $f(x)$ and $g(x)$. Will $f(g(x))$ ever equal $g(f(x))$ using the functions defined above? If so, when; if not, why not?

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52. Pay TV channels scramble their signals and sell customers boxes that unscramble the signals. If the signal comes in as a smooth curve, the set provides the program in viewable condition. A scrambled program is not a smooth curve. The box converts the signal to a smooth curve ($y = \sin 3x$ is an example of a smooth curve); $y = 2\sin x \cos x + \sin 3x$ is an example of a curve that is not smooth enough for reception without being unscrambled. How can $y = 2\sin x \cos x + \sin 3x$ be translated to a smooth curve that will give good reception of the program?[2]

EXTENSION: Describe other examples that could be used as signal scramblers.

[2]Initial discussion provided by Dr. Irwin Hoffman, Englewood, CO

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53. Use a graphing calculator or software to show $y^2 = (\sin x + x)^2$ and $y^2 = (3\sin x + x)^2$. After graphing these two equations, use a different color if possible, to graph $y^2 = ((\cos 10x + 2)(\sin x) + x)^2$. The first two equations form an "envelope" for the third. Find or create two other examples showing a curve or curves that serve as an envelope for another curve or set of curves.[3]

[3]Initial discussion provided by Dr. Irwin Hoffman, Englewood, CO

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54. Use a graphing calculator or software that does symbolic manipulations to solve the following: I am thinking of two numbers. Their sum is thirteen. Three times the first minus four times the second is negative five. What are the numbers?

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55. Is the following a continuous function? Explain your answer.[4]

$y = 2x^4 - 3x^2 + 1$
$y = 3x^4 - x^2 + x - 1$

EXTENSION: Find similar examples and explain the solutions.

[4]Initial discussion provided by Dr. Donald Danner, Provo, UT

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STATEMENT AND VALUE CARDS

56. On the following pages are sets of "statement cards" (pp. 40–50, containing polynomials) and "value cards" (pp. 51–54, containing values to be substituted into the polynomials). There are five sets of reproducible cards. Each set of cards is marked with a particular symbol. This symbol is used to indicate which value cards go with each set of statement cards.

To make long-lasting card sets, copy the pages and back them with construction paper or other heavy paper, then cut out the statement cards and value cards.

RULES: Make 2 decks of cards: statement cards and value cards. Deal the same number of statement cards to each player. Place the value cards in a stack face down on the table. Turn over the top value card. Each player substitutes the number on the value card in each statement card held. Each person plays one statement card on a trick; the highest card using the given value takes the trick. Turn over the next value card and repeat the process. The person taking a total of 5 tricks wins (statement and value cards may need to be reshuffled).

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$n-3$	$n+7$	$n+6$	$n+23$	$n+54$
$n-1$	$n+5$	$n+11$	$n+17$	$n+31$

$n-3$	$n+7$	$n+6$	$n+23$	$n+54$
$n-1$	$n+5$	$n+11$	$n+17$	$n+31$

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$n + 28$	$n + 48$	$37 + n$	$29 + n$	$33 + n$
$n + 35$	$42 + n$	$49 + n$	$21 + n$	$25 + n$

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$19 + n$	$47 + n$	$39 + n$	$4\frac{1}{2}x$	$4\frac{1}{2}x + 2\frac{1}{2}x$
$0 + n$	$63 + n$	$55 + n$	$2\frac{1}{2}x$	$6\frac{1}{4}x$

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$3x + 2\frac{1}{3}x$

$3x + 2\frac{1}{3}x$

$6\frac{1}{2}x + 2x$

$6\frac{1}{2}x + 2x$

$2\frac{1}{2}x + 7x$

$2\frac{1}{2}x + 7x$

$16x - 3\frac{1}{2}x$

$16x - 3\frac{1}{2}x$

$5x + 2\frac{1}{2}x - 1\frac{1}{2}x$

$5x + 2\frac{1}{2}x - 1\frac{1}{2}x$

51

$4\frac{1}{3}x + 2\frac{1}{2}x\ 4\frac{5}{6}x$

$4\frac{1}{3}x + 2\frac{1}{2}x\ 4\frac{5}{6}x$

$4\frac{1}{2}x + 3\frac{1}{4}x + 2\frac{1}{2}x$

$4\frac{1}{2}x + 3\frac{1}{4}x + 2\frac{1}{2}x$

$2\frac{1}{4}x + 7\frac{1}{2}x - 4\frac{3}{4}x$

$2\frac{1}{4}x + 7\frac{1}{2}x - 4\frac{3}{4}x$

$6\frac{1}{4}x - 2\frac{3}{4}x - 1\frac{3}{4}x$

$6\frac{1}{4}x - 2\frac{3}{4}x - 1\frac{3}{4}x$

$2\frac{1}{4}x + 2\frac{1}{4}x\,2\frac{1}{4}x + 2\frac{1}{4}x$

$2\frac{1}{4}x + 2\frac{1}{4}x\,2\frac{1}{4}x + 2\frac{1}{4}x$

50

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$4.2x + 3.4x$	$3x + 2.4x - 4.2x$	$(x + y)^2$	$x^2 + 4x + 4$	
$6.3x + 2.1x$	$27x - 10x - 9x$	$(x - y)^2$	$x^2 - 4xy + 4y^2$	
$64x - 60x$	$4x + 3.2x$			

$4.2x + 3.4x$	$3x + 2.4x - 4.2x$	$(x + y)^2$	$x^2 + 4x + 4$	
$6.3x + 2.1x$	$27x - 10x - 9x$	$(x - y)^2$	$x^2 - 4xy + 4y^2$	
$64x - 60x$	$4x + 3.2x$			

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$x^2 - y^2$	$81x^4 - 9x^2$	$x^2 + 4xy - 4y^2$	$x^2 + 2xy + y^2$	$4x^2 + 8xy + 9y^2$
$64x^2 - 32xy + 4y^2$	$100x^2 + 40xy + 4y^2$	$x^6 - y^6$	$x^2 - 2xy + y^2$	$9x^2 - 18xy + 9y^2$

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$2x^2 - 20xy + 25y^2$	$25x^2 + 100x + 100$	$(4x)^2 - 72xy + (9y)^2$	$z + (2z + 1)$	$3z - (2 - 2z)$
❀	❀	❀	❄	❄
$2x^2 - 20xy + 25y^2$	$25x^2 + 100x + 100$	$(4x)^2 - 72xy + (9y)^2$	$z + (2z + 1)$	$3z - (2 - 2z)$

$(2x)^2 - 20xy + 25y^2$	$4x^2 + 9y^2$	$36x^2 + 84xy + 40y^2$	$z - (z - 1)$	$5z + (4 - z)$
❀	❀	❀	❄	❄
$(2x)^2 - 20xy + 25y^2$	$4x^2 + 9y^2$	$36x^2 + 84xy + 40y^2$	$z - (z - 1)$	$5z + (4 - z)$

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$10 - (5 + z)$	$2z + (-z + 1)$	$5z - (2z + 4)$	$4(z - 2) - 5(2z + 3)$	$3z - 5(z - 1) + 6$
✲	✲	✲	✲	✲
$(z + 5) - 10$	$2z + (-z + 1)$	$5z - (2z + 4)$	$4(z - 2) - 5(2z + 3)$	$3z - 5(z - 1) + 6$
$(z^2) - (z^2 + 6)$	$-z - (z - 3)$	$z(z + 1) - (z^2 - 5z)$	$2z - 4 - (z + 4)$	$(2z + 3) + (z - 5) - 2(2z - 7)$
✲	✲	✲	✲	✲
$(z^2) - (z^2 + 6)$	$-z - (z - 3)$	$z(z + 1) - (z^2 - 5z)$	$2z - 4 - (z + 4)$	$(2z + 3) + (z - 5) - 2(2z - 7)$

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$z^2(z-1) - z(z^2+2z-4)$

❄

$z(z^2+4) + z(z-5) + 6$

$4z - (2z^2+3z+6) - (4z^2-5z)$

❄

$z^2(z-3) - z(z-3) + 4$

$4z^2 - 3(z^2-2z-5)$

❄

$5z^2 - (3z-2)^2 - 7(4z-6)$

$(m+1)(m-1)$

☆

$(2m-3)(3m-2)$

$(5m+3)(m-5)$

☆

$(m^2-m+1)(m+3)$

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$(m^2 - 3m + 2)(2m + 3)$

☆

$(3m + 5 - m^2)(m + 3)$

$(m + 2)(m^2 - 2m + 4)$

☆

$(-2m + m^2 + 3) \times (m - 2)$

$(4m - 6)(0.5m + 1)$

☆

$(m + 2)(m^2 - 2m + 4)$

☆

$(-2m + m^2 + 3)$

$(3m + 5 - m^2)(m + 3)$

☆

$2m(m - 2) - 4m(2m - 1)$

$(7m^2 - 3m + 1) \times (2m + 3)$

☆

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$(m^2 + m + 1) \times$ $(m^2 - m + 1)$ ☆ $(m^2 - m + 1) \times$ $(m^2 + m + 1)$	$(m^2 - m + 1) \times$ $(2 - m -)m^2$ ☆ $(m^2 - m + 1) \times$ $(2 - m -)m^2$	$(2m^2 + 3m)$ ☆ $(2m^2 + 3m)$	$(2m - 1)(3m + 2) -$ $(m - 3)(2m - 5)$ ☆ $(2m - 1)(3m + 2) -$ $(m - 3)(2m - 5)$	$(4m - 3)(m^2 + 1) +$ $(m^2 - 1)(2m - 7)$ ☆ $(4m - 3)(m^2 + 1) +$ $(m^2 - 1)(2m - 7)$
$(m + 1)^3$ ☆ $(m + 1)^3$	$(3m^2 - 4 - 3m)$ ☆ $(3m^2 - 4 - 3m)$	$(3m^2 - 4m + 3) \times$ $(2 - m)$ ☆ $(3m^2 - 4m + 3) \times$ $(2 - m)$	$(m - 1)(2m - 1) -$ $(3m + 1)(m - 2)$ ☆ $(m - 1)(2m - 1) -$ $(3m + 1)(m - 2)$	$3m(m^2 + 5m - 6) +$ $(m^2 + 1)(m - 1)$ ☆ $3m(m^2 + 5m - 6) +$ $(m^2 + 1)(m - 1)$

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$n = 5$	$n = 47$	$n = 16$	$n = 28$	$x = 12$
$n = 23$	$n = 158$	$n = 31$	$n = 96$	$x = 24$

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$x = 36$

$x = 36$

$x = 60$

$x = 60$

$x = 84$

$x = 84$

$x = 4,\ y = 1$

$x = 4,\ y = 1$

$x = 1,\ y = 2$

$x = 1,\ y = 2$

$x = 48$

$x = 48$

$x = 72$

$x = 72$

$x = 96$

$x = 96$

$x = 3,\ y = 8$

$x = 3,\ y = 8$

$x = -2,\ y = 3$

$x = -2,\ y = 3$

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$x = 2, y = 3$	$x = 1, y = -2$	$z = -2$	$z = 3$	$z = 6$
$x = -2, y = -5$	$x = 4, y = 2$	$z = -3$	$z = -1$	$z = 2.5$

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$z = -5$

$z = -5$

$m = -1$

$m = -1$

$m = -2$

$m = -2$

$m = -3$

$m = -3$

$m = -4$

$m = -4$

$z = 7\frac{4}{10}$

$z = 7\frac{4}{10}$

$m = 1$

$m = 1$

$m = 2$

$m = 2$

$m = 3$

$m = 3$

$m = 5$

$m = 5$

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SOLUTIONS

1. Let the three die values be x, y, and z. Select x. Multiplying by 2 gives $2x$.

$2x + 5$

$5(2x + 5) = 10x + 25$

$10x + 25 + y$

$10(10x + 25 + y) = 100x + 250 + 10y$

$100x + 250 + 10y + z$

$100x + 250 + 10y + z - 250 = 100x + 10y + z$, which is the expanded notation form of xyz

2. Let x be the phone number and y be the age. It is assumed that most students in algebra are at least 10, but if someone less than 10 is in the class, the age would be shown with a zero in the tens place.

$2x$

$2x + 5$

$50(2x + 5) = 100x + 250$

$100x + 250 + y$

$100x + 250 + y + 365 = 100x + y + 615$

$100x + y + 615 - 615 = 100x + y$

NOTE: This exercise is quite similar to number one. However, unsuspecting students are often still intrigued and begin to realize the similarities between the two. This is amplified if a few weeks pass between presentations.

3. Zero. The 24th factor is $(x - x)$.

4. Let the number chosen be x. Doubling x gives $2x$.

$2x + 4$

$(2x + 4)/2 = x + 2$

$x + 2 - x = 2$

The statement "Subtract the original number" can be changed to "Subtract 2," and the final result will be the number selected by the student initially. Similarly, rather than multiplying by 2, adding 4, and dividing by 3, multiply by 3, add 12, and divide by 3. Getting students to this stage of flexibility is a big help in many classroom algebraic procedures.

5. WHICH = x and WHAT = $35x$

WHICH + WHAT = $x + 35x = 36x$

$36x/4 = 9x$

WHICH squared = x^2

$x^2 = 9x$

Dividing out common factors gives $x = 9$. Therefore, WHICH = 9 and WHAT = (35)(9) = 315.

6. $p + d = n$

$0.01p + 0.10d + 0.05(p + d) = 1.65$

$(p, d, n) = (0, 11, 11), (5, 9, 14), (10, 7, 17), (15, 5, 20), (20, 3, 23), (25, 1, 26)$

7. Remainder = 4. Primes greater than 3 are of the form $6n \pm 1$, where n is a counting number: $(6n \pm 1)^2 = 36n^2 + 12n + 1$. Add 15 and get $36n^2 + 12n + 16$. Divide by 12 and get a remainder of 4.

8. This leads to composition of functions.

$\triangle(x) = x + 3$

$\boxed{x + 3} = x^2 + 6x + 5$

$x^2 - \boxed{\triangle(x)} - 4 = 5$

Substituting, $x^2 - (x^2 + 6x + 5) - 4 = 5$

$-6x - 13 = 5$

$-6x = 14$

$x = -7/3$

9. $abcabc = abc(1000) + abc$
$abc(1000) + abc = 1001(abc)$
The factors of 1001 are 7, 11, and 13.

10. Let the selection be x.

$(x + 1)^2 - x^2 = x^2 + 2x + 1 - x^2$

$x^2 + 2x + 1 - x^2 = 2x + 1$

$2x + 1 - 1 = 2x$

$(2x)/2 = x$

This exercise works with both integers and rationals. When dealing with negatives, be careful with the signs. For example, select -7 as the initial number; -6 is the next highest number. The problem then is $(-6)^2 - (-7)^2 = -13$. Subtracting 1 gives -14; $(-14)/2 = -7$.

11. If x is the first number, its square is x^2. The next counting number is $x + 1$, and $(x + 1)^2 = x^2 + 2x + 1$. So take the initial square and add twice the original number plus one.

12. c = cost of less expensive

c + $6.00 is cost of other

$c + c + \$6.00 = \$6.80$

$2c = \$0.80$

$c = \$0.40$

13. Let $a - 1 = 1234567890$

$$\frac{a - 1}{a^2 - [(a - 1)(a + 1)}$$

$$\frac{a - 1}{a^2 - (a^2 - 1)} = a - 1$$

14. Let the three digits be x, y, and z. The six 2-digit numbers would be xy, xz, yx, yz, zx, zy. In expanded notation, the sum of all 6s would be $(10x + y) + (10x + z) + (10y + x) + (10y + z) + (10z + x) + (10z + y) = 22(x + y + z)$

15. Substituting x = 0 gives c or r; $x = 1$ gives $a + b + c$ or s; $x = 2$ gives $4a + 2b + c$ or t. Subtracting $s - r$ gives $(a + b + c) - c = a + b$ or u; $t - s$ gives $(4a + 2b + c) - (a + b + c) = 3a + b$ or v; $v - u$ gives $(3a + b) - (a + b) = 2a$ or w.

16. Dividing by $b - a$ is undefined.

17. Consider the number xy, where y represents the ones digit of the number, and x represents all other places (x may not be a single digit but it is a counting number). In expanded notation, $xy = 10x + y$, and 10 is always divisible by 2. If y is even, it can be expressed as $y = 2z$. Then $xy = 10x + 2z$. Simplifying $10x + 2z = 2(5x + z)$. Two is a factor of the original number, so the original number is divisible by 2. For 4, start with zbc where b and c are the tens and ones digits respectively, and z is any counting number, so expanded notation is $100z + bc$. If bc is a multiple of 4, the whole number is divisible by 4. With 8, use $1000r + stu$. Common characteristics of the divisibility rules for 2, 4, and 8 can be seen in the expanded notation: 10^1 for 2^1; 10^2 for 2^2; 10^3 for 2^3.

18. Let rst be any 3-digit number (any number of digits could be used).

$rst = 100r + 10s + t$

$100r + 10s + t = (99 + 1)r + (9 + 1)s + t$

$(99 + 1)r + (9 + 1)s + t = 99r + 9s + r + s + t$

In $99r + 9s + r + s + t$, 99 and 9 will always be divisible by 3 and so will any of their respective multiples. That leaves only $r + s + r$, which is the sum of the digits. If $r + s + t$ can be expressed as a multiple of 3, the original number has 3 as a factor and is divisible by 3. If $r + s + t$ is not a multiple of 3, then the original number does not have a factor of 3 and is not a multiple of 3. For the 9 rule, substitute 9 for 3 in the discussion above.

19. Doubling the amount each day in a 31-day-month gives a total of $21,474,936.48, with $10,737,418.24 on the

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last day. (This compares to $310,000 at $10,000/day.) A 30-day-month total is $10,737,418.24 with $5,368,709.12 on the last day. A 29-day-month total is $5,368,709.12 with $2,684,354.56 on the last day. A 28-day-month total is $2,684,354.56 with $1,342,177.28 on the last day. You would be $18,790,581.92 ahead in a 31-day versus a 28-day month.

20. The largest number is 9^{9^9} (nine raised to the ninth power raised to the ninth power), which is over 350,000,000 digits long. If the answer is typed using 10 digits per inch, the answer is over 550 miles long.

21. The correct interpretation of the equation is 2^{81}, which is 2,417,851,963,229,258,349,412,352

22. Let x be the first term. Let y be the second term. The ten terms of the sequence are x, y, $x + y$, $x + 2y$, $2x + 3y$, $3x + 5y$, $5x + 8y$, $8x + 13y$, $13x + 21y$, $21x + 34y$. The sum of these first ten terms is $55x + 88y$. Dividing out common factors gives $11(5x + 8y)$; $5x + 8y$ is the seventh term in the original sequence. The sum can be determined by multiplying the seventh term in the sequence by eleven.

23. Easy example $(0)(0) = 0 = 0 + 0$. If n is the first value in the pair, then dividing n by $(n - 1)$ will give the second value. For example, let $n = 7$. Then $n - 1 = 6$. The two values would be 7 and 7/6, and $(7)(7/6) = 49/6$; $7 + 7/6 = 42/6 + 7/6 = 49/6$.

24. To find the solution, consider $(x5)^2$, where x is any whole number. In expanded notation, $(x5)^2 = (10x + 5)^2$, and $(10x + 5)^2 = 100x^2 + 100x + 25$. Focus on the first two terms only and divide out the common factor of 100: $100x^2 + 100x = (x^2 + x)100$, which shows why the value can always end in 25.

Now consider $x^2 + x$. This will be one of two rules students should see: square the value to the left of the 5 and add it to itself (in 75^2, $7^2 = 49$, $49 + 7 = 56$, the rest of the answer to the left of 25 in 75^2). The other rule comes from factoring $x^2 + x$ to $x(x + 1)$ (in 65^2 $(6)(6 + 1) = (6)(7) = 42$).

25. Multiply the ones digits of the factors. That answer becomes the ones and tens digits of the final product. The rest of the final product is found by multiplying all values to the left of the ones digit (they must be the same in both factors) by itself plus one, for example, in (62)(68), (6)(7) = 42. Proof: Each of the factors is $x5 \pm y$, ($x5$ is read $x$ty-five like 75 is read seventy-five) where x is any counting number and $0 < y \leq 4$. In expanded notation

$(x5 + y)(x5 - y) = (10x + 5 + y)(10x + 5 - y)$

$(10x + 5 + y)(10x + 5 - y) = 100x^2 + 100x + 25 - y^2$

26. Let a, b, c, and d be the digits involved.

$(ab)(cd) = (10a + b)(10c + d) = 100ac + 10bc + 10ad + bd$

$(ba)(dc) = (10b + a)(10d + c) = 100bd + 10bc + 10ad + ac$

Setting the two expressions equal to each other and subtracting 10bc and 10ad from both sides gives $100ac + bd = 100bd + ac$. This can only be true if $ac = bd$.

27. Let n be the selected number.

$n + (n + 1)$

$n + (n + 1) + 9 = 2n + 10$

$(2n + 10)/2 = n + 5$

$(n + 5) - 5 = n$

It works for integers. Rationals will work in special cases but not in general.

28. Let a be the selected number. Then 6 times that number is $6a$.

$6a + 12$

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$(6a + 12) / 3 = 2a + 4$

$2a + 4 - 2 = 2a + 2$

$(2a + 2)/2 = a + 1$

$a + 1 - a = 1$

$1 + 9 = 10$

Integers and rationals give a sum of 10.

29. n terms and a sum of $(n + 1)$ for each term gives $[(n)(n + 1)]/2$

EXTENSION: Using $3 + 6 + 9 + \ldots + 300$ gives $3(1 + 2 + 3 + \ldots + 100)$. Starting with a number other than one means the sum of the values prior to the starting point would need to be subtracted.

30. In the first row, day 1 = x, day 2 = $x + 1$, day 3 = $x + 2$. In the next row, day 1 = $x + 7$, day 2 = $x + 8$, day 3 = $x + 9$. In the last row, day 1 = $x + 14$, day 2 = $x + 15$, day 3 = $x + 16$. The sum of all the dates = $9x + 72$; $9x + 72 = 9(x + 8)$. To get the sum quickly, add 8 to the given value and multiply that sum by 9. The 16-date square is $16(x + 12)$. A 25-date square cannot be found (see a 31-day month, using 1 as the first date). If it existed, a 25-date square would be $25(x + 16)$. The generalization becomes $n^2[x + (n - 1) \times 4]$, where n is the size of the square and x is the first value selected.

31. Deleting the last digit is akin to rewriting 2296 as 229[0] + 6 and ignoring the zero (shown by [0]). Doubling the last digit and subtracting is akin to subtracting 20 times that last digit: 229[0] + 6 = 217[0] + $\underline{20(6)+6}$. Looking only at the underlined part, $\underline{20(6) + 6} = 21(6)$ which is divisible by 7. Similarly, $2296 = 700 + 210(7) + 21(6)$, and 7 is a factor of each term, so the original number is a multiple of 7.

32. The resulting figure after the diagonal fold is an isosceles right triangle. The fold parallel to one leg may be made at any point on the other leg. The length of the sides perpendicular to a and b is $(a - b)$. Rectangle from "L":

Length $(a + b)$

Width $(a - b)$

Area $(a + b)(a - b) = a^2 - b^2$

33. Varies

34. The rectangle is constructed of the pieces $x^2 + 3x + 2$, which factors to $(x + 2)(x + 1)$. A polynomial that cannot produce a rectangle cannot be factored over the integers.

35. $4n + 6$, where n = the number of "L's."

36. First time, 4 pieces. Second time, 7 pieces. Third time 10 pieces. In general n repetitions gives $3n + 1$ pieces.

37. Paper 0.003 inches thick gives a stack 5.331×10^7 miles high. Paper 0.004 inches thick gives a 7.108×10^7 mile high stack. The easy way to get the second answer is $(4/3)(5.331 \times 10^7)$.

38. Area = $(11x)(15x - 7)$. The following values are all side lengths of the respective squares: 1 is given; x is the next largest square; $x + 1$ comes from lengths of two adjacent squares on the top side $(x) + 1$; $x + 2$ comes from two adjacent squares on the right side $(x + 1) + 1$; $x + 3$ comes from two adjacent squares on the bottom side $(x + 2) + 1$; $3x + 1$ comes from lengths of two adjacent squares on the bottom side $(2x + 1) + x$. The square with sides $3x - 3$ is probably the most difficult to determine. It comes from $(3x + 1) + (x - 1) - (x + 3)$. The square with sides $6x - 2$ comes from the lengths of the two squares on the bottom side $(3x - 3) + (3x + 1)$; $9x - 5$ comes from the lengths of two adjacent squares on the right side $(6x - 2) + (3x - 3)$; $2x + 5$ comes from the lengths of two adjacent squares on the right side $(x + 3) + (x + 2)$. The length of the rectangle is found by adding the lengths on the left $(11x)$ and top $(15x - 7)$ sides.

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39. The cards in each stack can be represented by $14 - x$, $14 - y$ and $14 - z$, and $(14 - x) + (14 - y) + (14 - z) = 42 - x - y - z$, the number of cards on the table. So, $52 - (42 - x - y - z) = 10 + x + y + z$. Revealing x and y and adding 10 will give the remaining card.

40. Let x = selected card, $x + 1$ = next counting number, and k = suit value. So $x + (x + 1) = 2x + 1$, and $5(2x + 1) = 10x + 5$. The value $10x + 5 + k$ is given to you. Subtract 5. The ones digit gives the suit and the tens or hundreds and tens gives the card.

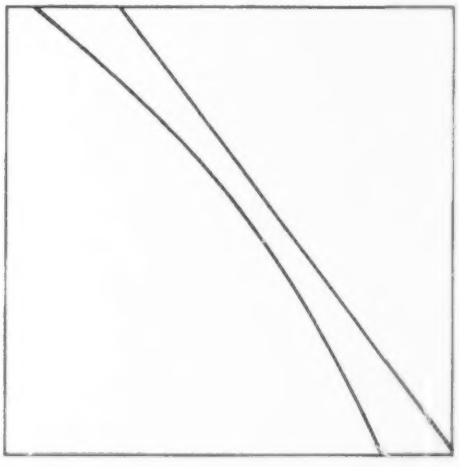
41. Suppose the entire trip was 100 miles. Averaging 50 mph, that trip would take 2 hours. Driving the first half of the trip at 25 mph uses the necessary 2 hours, so the task is impossible. There is no time left for returning.

42. x = log length; $x = 20 + (x/2)$; $x = 40$

43. Varies

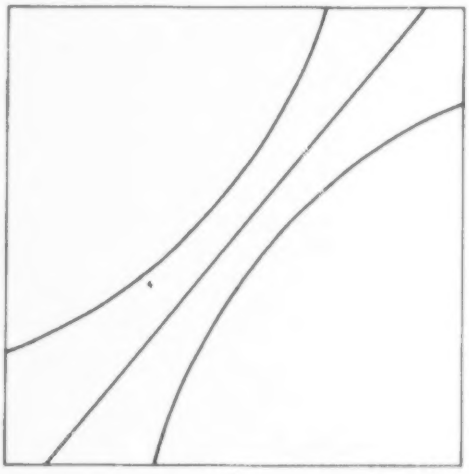
44. Almost any form of racing provides applications of timing vehicles through a distance.

45. Graph and zoom (shown below). Substituting gives $2x^2 - (98/5)x + 1621/25 = 16$. Solving this for x gives no solution over the real numbers and $x = \pm \frac{1}{10}\sqrt{41i} + \frac{49}{10}$ over the complex numbers.



46. The basic equation for a circle is $(x - h)^2 + (y - k)^2 = r^2$, where r is the radius of the circle and (h, k) is the point at the center of the circle. The equations for the two circles in this problem are $(x + 2)^2 + (y + 5)^2 = 4$ and $(x + 5)^2 + (y + 2)^2 = 4$. The equation for the line is $y = x$. The form in which you enter these equations will depend on the calculator or software that you use. Some graphing calculators may not be able to handle these equations, others may handle them only as programs in which values are substituted.

Graph and zoom (shown below). Use the coordinates of the centers in the formula for finding the distance between two points and get a value greater than 4. Since the radius of each circle is 2, a distance greater than 4 means the circles are not tangent.



47. With MISSISSIPPI, you get $MI^4P^2S^4$. With mississippi, get mp^2s^4 because $i^4 = 1$ since $i = \sqrt{-1}$.

48. The graph intersects the y-axis at the addend.

49. The graph is shifted left on the x-axis by the value of the addend.

50. The graph is shifted left on the x-axis by the value of the addend within the parentheses and upward on the y-axis by the amount of the other addend.

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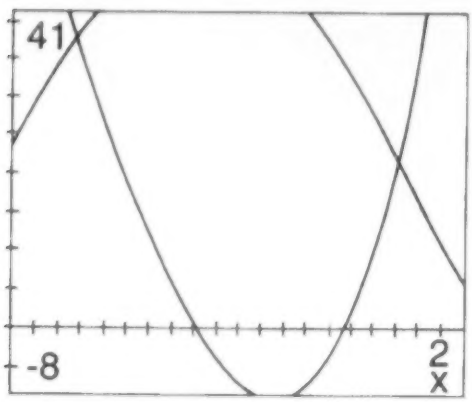
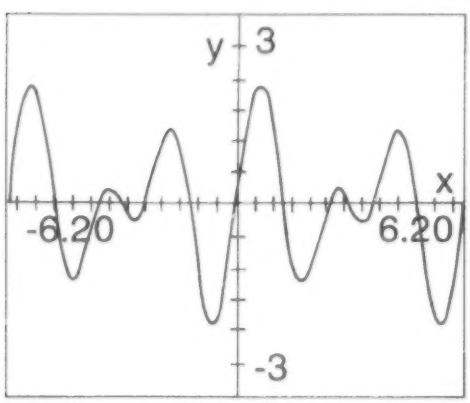
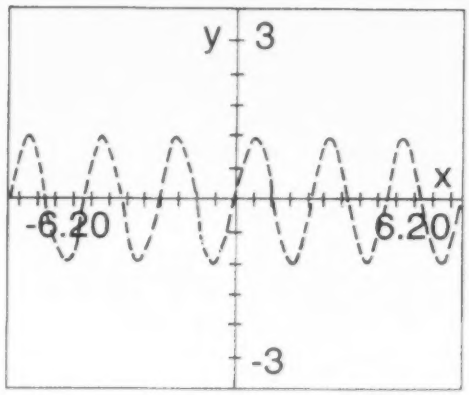
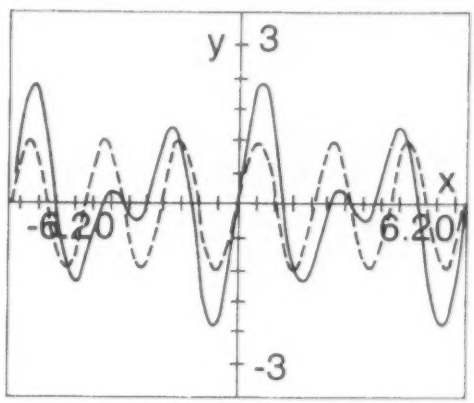
51. Graph $y = f(g(x)) = 3x^4 + 18x^3 - 2x^2 - 87x + 61$ and $y = g(f(x)) = 9x^4 - 30x^3 - 8x^2 + 55x + 24$. Use the zoom and trace functions to find the points of intersection. The curves intersect at (0.2701, 37.728) and (1.7076, 21.738).



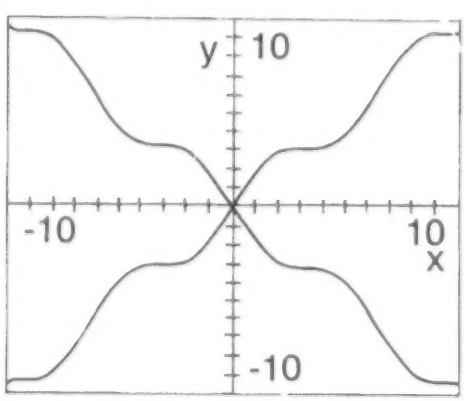
52. Note that $2\sin x \cos x + \sin 3x - \sin 2x = \sin 3x$ since $\sin 2x = 2\sin x \cos x$. The first graph below shows $2\sin x \cos x + \sin 3x$. The next graph shows $2\sin x \cos x + \sin 3x - \sin 2x$ (i.e., $\sin 3x$). The final graph shows the two curves together.







53. For many graphing calculators, the equations that you will actually need to enter are $y = \pm(\sin x + x)$; $y = \pm(3\sin x + x)$; and $y = \pm((\cos 10x + 2)(\sin x) + x)$. The graphs below show (1) $y^2 = (\sin x + x)^2$, (2) $y^2 = (3\sin x + x)^2$, (3) $y^2 = (\sin x + x)^2$ and $y^2 = (3\sin x + x)^2$; and (4) $y^2 = (\sin x + x)^2$, $y^2 = (3\sin x + x)^2$, and $y^2 = ((\cos 10x + 2)(\sin x) + x)^2$.



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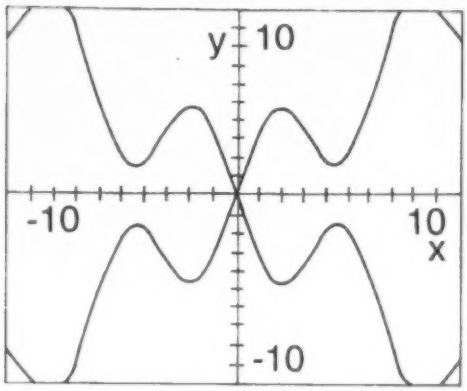
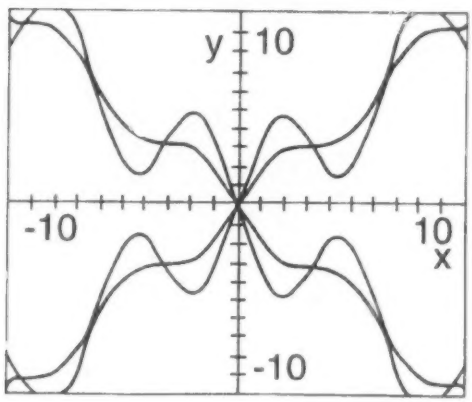
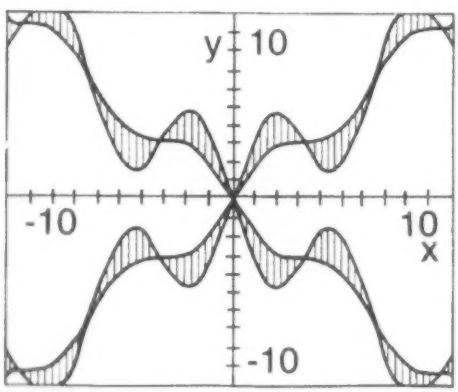
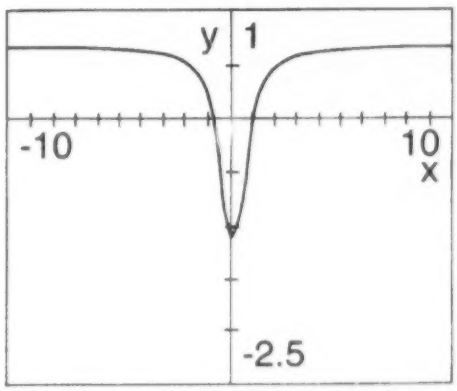


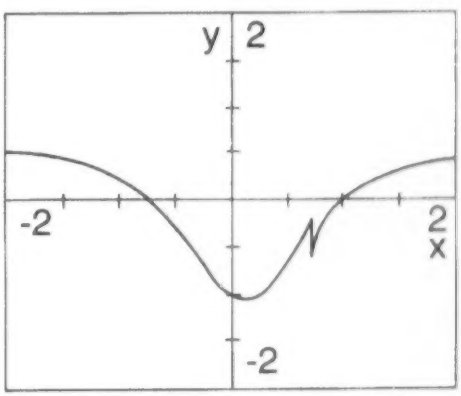


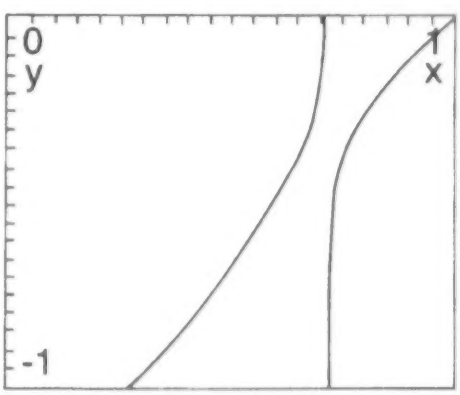


54. The two equations are $x + y = 13$ and $3x - 4y = -5$. For most graphing calculators, you will need to solve both equations for y before entering them: $y = 13 - x$ and $y = (3x + 5)/4$. The equations are solved by (6.7143, 6.2858).

55. If graphed using most graphing calculators or software, the function will appear to be continuous, when in fact it is not. Factoring shows a common factor of $(x + 1)$ indicating there is a hole in the graph at $x = -1$. Graphing software and calculators are not currently capable of showing a single missing point. Graphing the curve between $x = \pm 2$ and $y = \pm 2$ shows a small distortion in the curve around $x = 0.75$. Magnifying the curve in that area shows another break.







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